

**PRACTICE QUESTIONS ON BAYES'S FORMULA AND ON
PROBABILITY
(NOT TO BE HANDED IN)**

Note 1. On the exam, you will have to solve a question which is an application of Bayes's formula. In some of the following practice questions, the numbers might not come out very clean. However, on the exam, I will make sure that the numbers come out clean and simplify so that you can perform the computation to the end without a calculator to find the desired probability (assuming that you can do basic arithmetic, which I know you all can do).

If somehow in the end you get an answer to a computation of a probability which is not between 0 and 1 and leave it like that you will lose a lot of points! I expect you to know that the probability is a number between 0 and 1. So if somehow you get something else, that should be a sign you did something wrong. **Moral: Stare at your answer at the end and make sure it makes sense.** The same applies if I ask you to compute an area of a function whose graph is above the x-axis and you give as answer a negative number.

Note 2. Here are some observations:

- Please note that if you are told something like “5% of the population has this disease” this means that the probability of a person having the disease, which is the number of all people having the disease divided by the number of all people, is $\frac{5}{100} = .05$. If we denote by D the event that someone has the disease, $P(D) = .05$.
The probability is always a number between 0 and 1.
- Now if the probability of someone having the disease is .05, then the probability of someone not having the disease is .95 **because** the probability of D^c , i.e., the probability of event D not happening is $P(D^c) = 1 - P(D)$.
So this is why it was important that we showed this identity in the beginning; it comes up all the time.

EXERCISES

Exercise 1. Questions 39 and 40 from your homework are applications of Bayes's formula. You can also try 37 and 38.

Exercise 2. A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that $F \cup M = \Omega$, i.e., that there are no other maladies in that neighborhood.

A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $P(R | M) = .95$. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is $P(R | F) = 0.08$.

Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

Exercise 3. In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumors and 90% of benign tumors.)

95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do you agree?

Exercise 4. Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side is colored black.

The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Exercise 5. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%.

Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?